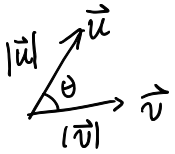
MATH 2E Prep: Products of Vectors

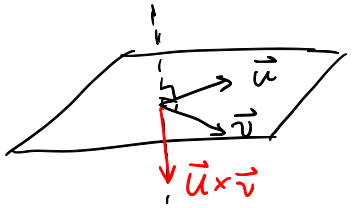
Facts to Know:

1. Dot product:
   - If $\vec{u} = \langle x_1, y_1, z_1 \rangle$, $\vec{v} = \langle x_2, y_2, z_2 \rangle$, then $\vec{u} \cdot \vec{v} =$ $x_1x_2 + y_1y_2 + z_1z_2$
   - $\vec{u} \cdot \vec{v}$ is a (scalar) / vector.
   - $\vec{u} \cdot \vec{v} =$ $|\vec{u}| \cdot |\vec{v}| \cdot \cos\theta$



   - $\vec{u} \perp \vec{v}$ if and only if $\vec{u} \cdot \vec{v} = 0$.

2. Cross product:
   - If $\vec{u} = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$, $\vec{v} = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k}$, then $\vec{u} \times \vec{v} =$ $\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$
   - $\vec{u} \times \vec{v}$ is a scalar / (vector).
   - $|\vec{u} \times \vec{v}| =$ $|\vec{u}| \cdot |\vec{v}| \cdot \sin\theta$
   - $\vec{u} \times \vec{v}$ is perpendicular to both $\vec{u}$ and $\vec{v}$.



   - The right-hand rule.
   - $\vec{u} \parallel \vec{v}$ if and only if $\vec{u} \times \vec{v} = \vec{0}$.

Examples:

1. Determine whether $\vec{u} = \langle 3, 5, 4\rangle$ is perpendicular to $\vec{v} = \langle 1, -2, 2\rangle$.

$\vec{u}\cdot\vec{v} = 3\cdot 1 + 5\cdot(-2) + 4\cdot 2 = 3 - 10 + 8 = 1 \neq 0$

$\Rightarrow$ Not $\perp$.

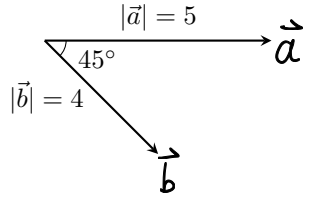
2. $\vec{u} = \langle 3, 5, 4\rangle$, $\vec{v} = \langle 1, -2, 2\rangle$, compute $\vec{u} \times \vec{v}$ and $\vec{v} \times \vec{u}$.

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 5 & 4 \\ 1 & -2 & 2 \end{vmatrix} = \vec{i}\begin{vmatrix} 5 & 4 \\ -2 & 2 \end{vmatrix} - \vec{j}\begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} + \vec{k}\begin{vmatrix} 3 & 5 \\ 1 & -2 \end{vmatrix}$$

$$= 18\vec{i} - 2\vec{j} + (-11)\vec{k} = \langle 18, -2, -11\rangle$$

$$\vec{v}\times\vec{u} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -2 & 2 \\ 3 & 5 & 4 \end{vmatrix} = -\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 5 & 4 \\ 1 & -2 & 2 \end{vmatrix} = -\vec{u}\times\vec{v} = \langle -18, 2, 11\rangle$$

3. Use the following figure to find $\left|\vec{a} \times \vec{b}\right|$, and determine whether $\vec{a} \times \vec{b}$ is directed into the page or out of the page.



$|\vec{a}\times\vec{b}| = |\vec{a}|\cdot|\vec{b}|\cdot\sin(45°) = 5\cdot 4\cdot\frac{\sqrt{2}}{2} = 10\sqrt{2}$

$\vec{a}\times\vec{b}$ is into the page.